1.1 The Limit of a Function (1.4)

(now start taking notes!)

One-Sided Limits

$\lim_{x \to a^-} f(x)$

- the limiting value of f(x) as x approaches "a" from the left (aka "left-hand limit")

$\lim_{x \to a^+} f(x)$

- the limiting value of f(x) as x approaches "a" from the right (aka "right-hand limit")

Two-Sided Limit

$\lim_{x \to a} f(x)$

- the limiting value of f(x) as x approaches "a" from the left AND the right
- if the left and right limits are different then $\lim_{x \to a} f(x)$ does not exist
- $\pm \infty$ are not considered "limiting values"

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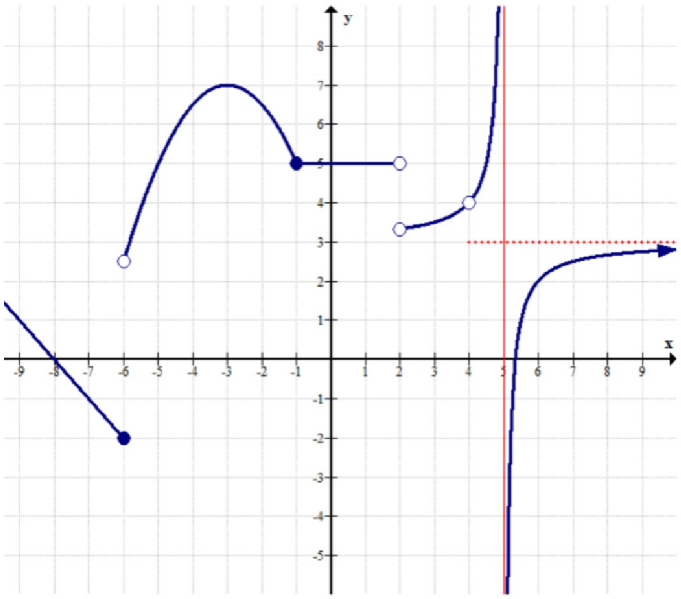
Limits from a Graph:

Eg.1 Evaluate the following limits from the graph of y=f(x) shown below:



$\lim_{x\to -6^-} f(x)$ $\lim_{x\to -6^+} f(x)$ $\lim_{x\to -6} f(x)$

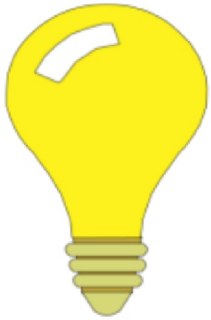
$\lim_{x\to -1^-} f(x)$ $\lim_{x\to -1^+} f(x)$ $\lim_{x\to -1} f(x)$

$\lim_{x\to 2^-} f(x)$ $\lim_{x\to 2^+} f(x)$ $\lim_{x\to 2} f(x)$

$\lim_{x\to 4^-} f(x)$ $\lim_{x\to 4^+} f(x)$ $\lim_{x\to 4} f(x)$

$\lim_{x\to 5^-} f(x)$ $\lim_{x\to 5^+} f(x)$ $\lim_{x\to 5} f(x)$

Think About:



Can the limit at "a" exist even if the function does not exist at "a" ?

yes. hole at x=a (x=4 above)

Is it possible for the function to exist at "a" but not the limit at "a" ?

yes. piecewise that is discontinuous but defined at "a" (x=-6 above)

Can the limit at "a" be the same as the function value at a?

yes. function is continuous at "a".... the point (a, f(a)) is on the graph (x=-1 above)

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Limits from Equations:

Eg. 2 Evaluate each limit. If the limit does not exist, explain why.

a) $\lim_{x\to 2} 3x^2 - 5x + 1$

b) $\lim_{x\to 3^+} \sqrt{x-3}$

c) $\lim_{x\to 2^-} \sqrt{x-2}$

d) $\lim_{x\to 0} \sqrt{x^2+5}$

e) $\lim_{x\to 3} \frac{1}{2x-1}$

f) $\lim_{x\to 7^-} -3x+1$

g) $\lim_{x\to 5^+} \frac{1}{x-5}$

h) $\lim_{x\to 5^-} \frac{1}{x-5}$

i) $\lim_{x\to 5} \frac{1}{x-5}$

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Piecewise Functions

Eg. 3 Given y = g(x), evaluate the indicated limits.

$$g(x)=\begin{cases}2x+1, & x\le 2\\ x^2+1, & 2<x\le 5\\ 5-x, & x>5\end{cases}$$

a) $\lim_{x\to 2} g(x)$

b) $\lim_{x\to 5} g(x)$

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