1.2 Properties of Limits (1.5)

For any real number "a", f and g are functions that have limits at x=a:

What does the function look like?

1. $\lim\limits_{x \to a} k =$ k, for any constant k

2. $\lim\limits_{x \to a} x =$ a

3. $\lim\limits_{x \to a} \left[f(x) \pm g(x) \right] = \lim\limits_{x \to a} f(x) \pm \lim\limits_{x \to a} g(x)$

4. $\lim\limits_{x \to a} \left[c \cdot f(x) \right] = c \left[\lim\limits_{x \to a} f(x) \right]$, for any constant c

5. $\lim\limits_{x \to a} \left[f(x) g(x) \right] = \left[\lim\limits_{x \to a} f(x) \right] \left[\lim\limits_{x \to a} g(x) \right]$

6. $\lim\limits_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim\limits_{x \to a} f(x)}{\lim\limits_{x \to a} g(x)}$, provided that $\lim\limits_{x \to a} g(x) \neq 0$

7. $\lim\limits_{x \to a} \left[f(x) \right]^n = \left[\lim\limits_{x \to a} f(x) \right]^n$, for any rational number n

Eg. 1 Use the properties of limits to evaluate.

a) $\lim\limits_{x \to -2} \left(2x^3 - 7x + 4\right)$

b) $\lim\limits_{x \to 1} \left[-2\left(x-3\right)^2 + 4\right]$

If $f(x)$ is a polynomial, then $\lim\limits_{x \to a} f(x) = f(a)$

c) $\lim\limits_{x \to 3} \dfrac{3x^2 - x + 4}{2x - 3}$

d) $\lim\limits_{x \to 8} \sqrt[3]{5x^2 - 18x - 8}$

- In all of the above cases the limit can be found by direct substitution... the function is continuous at the limit value so $\lim\limits_{x \to a} f(x) = f(a)$
- When direct substitution of x = a results in $\frac{0}{0}$ this is called an indeterminate form.
- When this happens we look for an equivalent function that has all the same values as f(x) except at x=a.

When direct substitution fails try:

1. Factoring

a) $\lim_{x \to 3} \frac{x^2 + x - 12}{2x^2 - 5x - 3}$

b) $\lim_{x \to 2} \frac{x^3 - 8}{x - 2}$

Jan 31-6:25 PM

3. Change of Variable

clone

a) $\lim\limits_{x\to 1}\dfrac{x-1}{\sqrt{x}-1}$

(yes, we could do this by rationalizing or forcing it to factor, but you need to see how to change the variable with a simple example)

Jan 31-7:17 PM

4. Consider Cases

$$\lim_{x \to -3} \frac{|x+3|(x+1)}{(x+3)}$$

graph of function

5. Think / Reason it out...

(picture the graph...draw from your range of knowledge of functions...)

$$\lim_{x \to 2} \sqrt{x^2 - 4}$$

Homework
Page 45
#4, 7-10, 13-16