1.2 Properties of Limits (1.5)

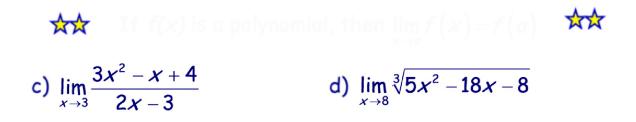
For any real number "a", f and g are functions that have limits at x=a:

- 1. $\lim_{k \to a} k = k$, for any constant k
- $2. \quad \lim_{x \to a} x = a$
- 3. $\lim_{x \to a} \left[f(x) \pm g(x) \right] = \lim_{x \to a} f(x) \equiv \lim_{x \to a} g(x)$
- 4. $\lim_{x \to a} \left[c \cdot f(x) \right] = c \lim_{x \to a} f(x) \quad \text{for any}$
- 5. $\lim_{x \to a} \left[f(x) g(x) \right] = \lim_{x \to a} f(x) \lim_{x \to a} g(x)$
- 6. $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f(x)}{g(x)}$, provided that $\lim_{x \to a} g(x) \neq 0$
- 7. $\lim_{x \to a} \left[f(x) \right]^n = \left[\lim_{x \to a} f(x) \right]^n$, for any rational number n



Eg. 1 Use the properties of limits to evaluate.

a) $\lim_{x \to -2} (2x^3 - 7x + 4)$ b) $\lim_{x \to 1} [-2(x - 3)^2 + 4]$



- So in all of the above cases the limit can be found by direct substitution... the function is continuous at the limit value so $\lim_{x \to \infty} f(x) = f(a)$
- When direct substitution of x = a results in $\frac{0}{0}$ this is called an indeterminate form.
- When this happens we look for an equivalent function that has all the same values as f(x) except at x=a.

When direct substitution fails try:

1. Factoring a) $\lim_{x \to 3} \frac{x^2 + x - 12}{2x^2 - 5x - 3}$ b) $\lim_{x \to 2} \frac{x^3 - 8}{x - 2}$ 3. Change of Variable

a) $\lim_{x \to 1} \frac{x-1}{\sqrt{x}-1}$ (yes, we could do this by rationalizing or forcing it to factor, but need to see how to change the variable with a simple example) (yes, we could do this by rationalizing or forcing it to factor, but you

Clone clone

4. Consider Cases

$$\lim_{x \to -3} \frac{|x+3|(x+1)}{(x+3)}$$



5. Think / Reason it out...

(picture the graph...draw from your range of knowledge of functions...)

$$\lim_{x\to 2}\sqrt{x^2-4}$$

Homework Page 45 #4, 7-10, 13-16



© Mark Parisi, Permission required for use.