1.4 Slope of the Tangent and Rate of Change

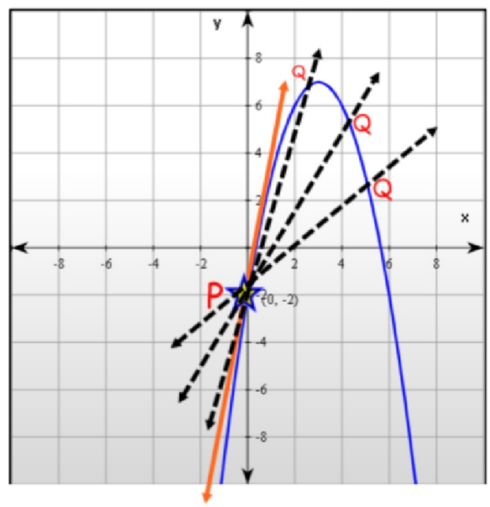
Recall: The slope of a line

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Since two points are needed, we use secants with point Q sliding closer towards point P.

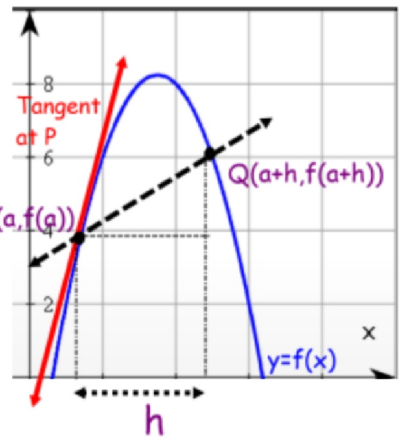
Tangent-is a straight line that most resembles the shape of graph at that point.

- The slope represents how steep the graph is at that point of tangency.



Desmos Example
https://www.desmos.com/calculator/p1vguu8m40

So the slope of a tangent to a curve at a point P is simply the limit of the slopes of the secants PQ as Q moves closer to P.



Let P(a,f(a)) be a fixed point on the graph.

Let Q be a point that is a horizontal distance of h units away

Slope of the secant above is:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{f(a+h) - f(a)}{a + h - a}$$

$$= \frac{f(a+h) - f(a)}{h}$$

Ex.1 a) Find the slope of the tangent to the curve $y = 2x^3$ at the point $x = 1$.
b) Determine the equation of the tangent.

Ex.2 Find the slope of the tangent to the curve $y = \sqrt{x-3}$ at the point (12,3).

Recall:

Average rate of change (slope of the secant)

$$\frac{\Delta y}{\Delta x} = \frac{f(a+h)-f(a)}{h}$$

Instantaneous rate of change (slope of the tangent)

$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

Average velocity using the position function s(t) from x=a to x=a+h:

$$\frac{\Delta s}{\Delta t} = \frac{s(a+h)-s(a)}{h}$$

Instantaneous Velocity using the position function s(t) at time

$$v(a)=\lim_{\Delta t\to 0}\frac{\Delta s}{\Delta t}=\lim_{h\to 0}\frac{s(a+h)-s(a)}{h}$$

Ex.3 Given the position function $s(t)=\frac{t}{t+1}$

a) Find the average rate of change for the interval $1\leq t\leq 3$

b) Find the velocity at t=4

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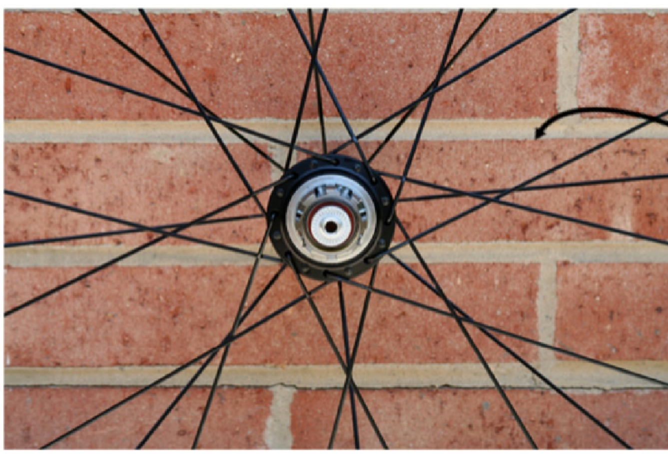
Homework
Pg. 18 # 8b,9a,10c,11f,15
Pg. 29 # 9,11,13,14,17



Can you spot the secants vs. tangents?

Jan 31-5:17 PM