

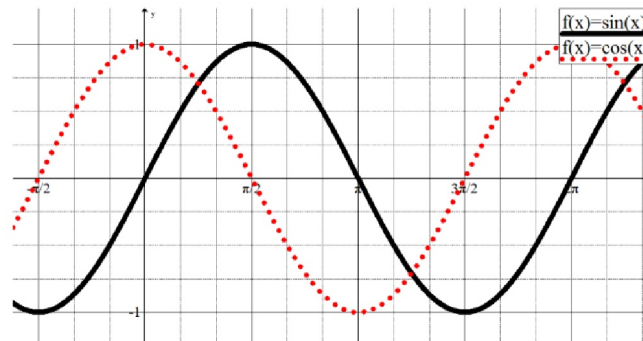
# 1.8 $e^x$ & $\ln x$



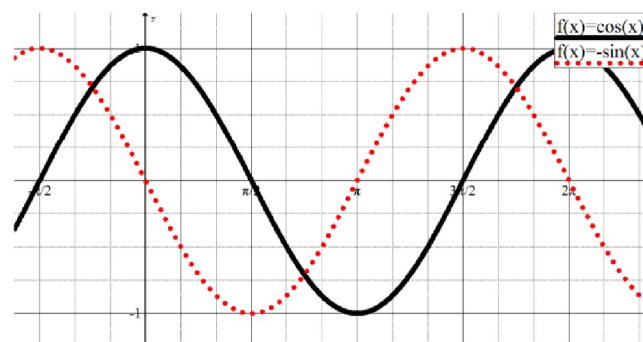
## Summary from Investigating Trig and Exp Functions:

IF  $f(x) = \sin x$  then  $f'(x) = \cos x$

link to 1.7  
investigation  
answers



IF  $f(x) = \cos x$  then  $f'(x) = -\sin x$



### Effect of Transformations on Base Trig Functions:

vertical translation - no effect on derivative

horizontal translation- derivative has same horizontal translation

vertical stretch/reflection - derivative has same vertical stretch/reflection

**\*\*horizontal stretch/reflection**

-derivative has the same transformations BUT ALSO impacts the vertical stretch of the derivative function

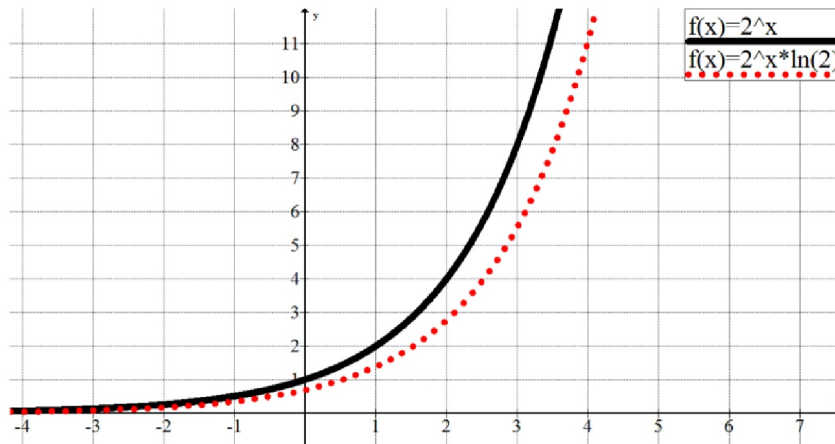
-a shorter period results in steeper tangents, a longer period results in flatter tangents...this affects the amplitude of the derivative function

IF  $f(x) = 2^x$ , then  $f'(x) = 2^x (k)$ , where  $k = \ln 2$ .  
 This represents a vertical **compression** of  $f(x)$ .

$$k = \frac{f'(x)}{f(x)}$$

$$= \ln 2$$

$$= \underline{\hspace{2cm}}$$

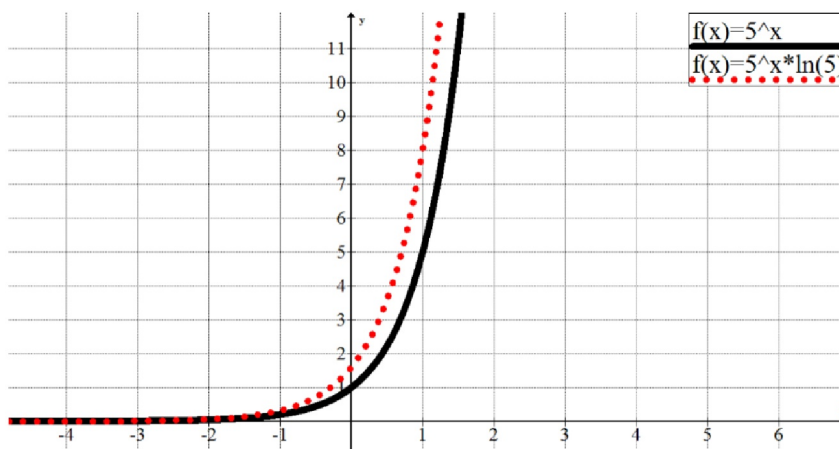


IF  $f(x) = 5^x$ , then  $f'(x) = 5^x (k)$ , where  $k = \ln 5$ .  
 This represents a vertical **stretch** of  $f(x)$ .

$$k = \frac{f'(x)}{f(x)}$$

$$= \ln 5$$

$$= \underline{\hspace{2cm}}$$



Q: If  $f(x) = a^x$ , is there a value of "a" where the derivative function would result in the same curve as the original function?  
(ie. no stretch or compression)

[Link to Graph File](#)

[Link to Document Player](#)

Graph of  $f(x)=e^x$

When "a" is approximately equal to \_\_\_\_\_, the derivative function is equivalent to the original function.

When  $f(x)=2.72^x$ , the derivative function  $f'(x)=2.72^x$  (same as  $f(x)$ )  
When the base is close to 2.72 the value of k approaches 1 ...therefore, there is no compression or stretch.

*Called Euler's Number  
(thus the "e").*

This is one of many definitions of the number "e".

$e$  = the base of an exponential function whose derivative function is itself

$e = 2.718281828459045235360287471352662497757247093699959574966\dots$

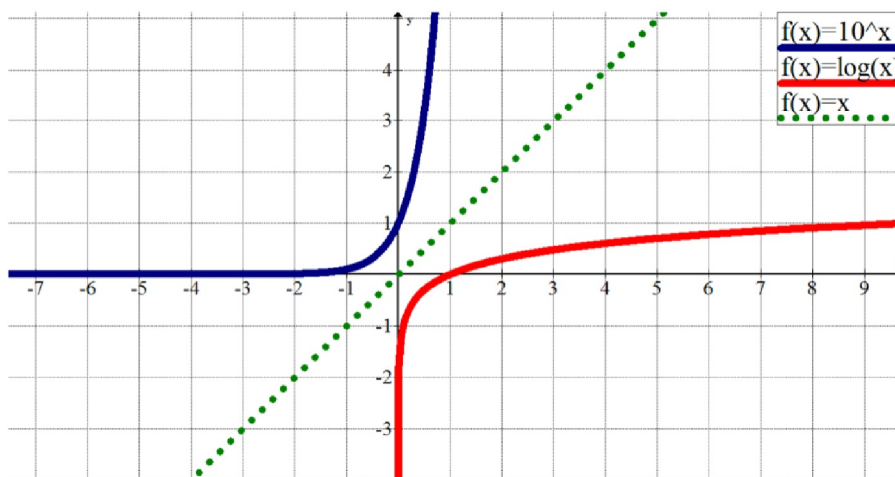
$e$  is an irrational number (it cannot be written as a fraction, it never ends, there is no repeating pattern)

Recall: The inverse of  $y=a^x$  can be written:

$$x = a^y \text{ (exponential form)}$$

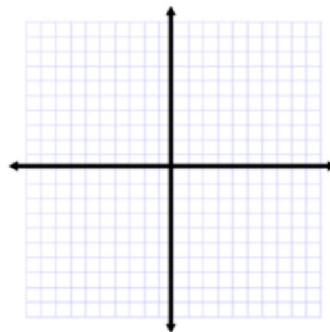
$$y = \log_a x \text{ (logarithmic form)}$$

Graphically:  $y = a^x$  and  $y = \log_a x$  are reflections in the line  $y=x$ .

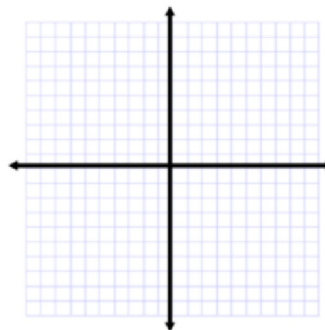


Ex. 1 Determine the inverse of each function.  
Graph  $f(x)$  and  $f^{-1}(x)$ .

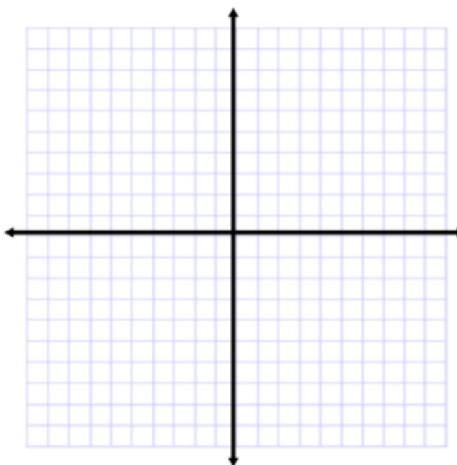
a)  $f(x) = 3^x$



b)  $f(x) = \left(\frac{1}{2}\right)^x$



c)  $f(x) = e^x$



$\log_e x$  is also known as  $\ln x$   
 $\log_e x = \ln x$   
 $\ln x$  is a log with base equal to  $e$   
 $\therefore \ln x$  is the inverse of  $e^x$

$\ln$  is also known as the natural log



## Log Laws:

$$\begin{aligned} \log_a m + \log_a n &= \log_a mn \\ \log_a \left( \frac{m}{n} \right) &= \log_a m - \log_a n \\ \log_a m^p &= p(\log_a m) \\ \log_a 1 &= 0 \\ \log_a a^x &= x \\ a^{\log_a x} &= x \end{aligned}$$

## Change of Base Formula:

$$\begin{aligned} \text{recall: } \log_b a &= \frac{\log_m a}{\log_m b} \\ \text{when } m &= e, \\ \log_b a &= \frac{\log_e a}{\log_e b} \\ &= \frac{\ln a}{\ln b} \end{aligned}$$

Ex. 2 Simplify/evaluate each of the following.

a)  $\log_5 1$       b)  $\log_6 6^x$       c)  $6^{\log_6 x}$

d)  $\ln e$       e)  $\ln 1$       f)  $e^{\ln x}$       g)  $\ln e^x$

Ex. 3 Simplify and evaluate each of the following.

a)  $\log_6 2 + \log_6 3$       b)  $\log_2 24 - \log_2 \left( \frac{3}{4} \right)$

c)  $2\log_2 \sqrt{8} - 2\log_2 4$       d)  $3\ln 2 - 3\ln 5$

Ex. 4 Use your calculator to evaluate.

a)  $\log_2 18$       b)  $\log_5 3$       c)  $\log_e 10$

# Homework: Handout



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