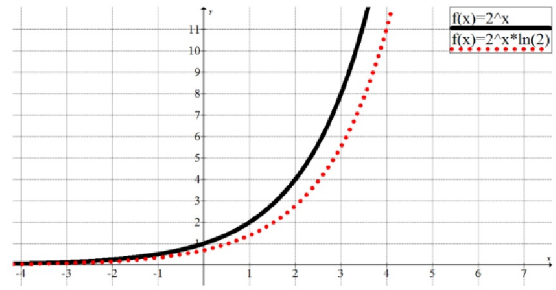
1.9 Limits and Derivatives of Exponential Functions

Consider the function $f(x) = 2^x$.

Through investigation we have determined that $f'(x) = 2^x(\ln 2)$, where $(\ln 2)$ is a constant.



(...and we're not totally sure what this "ln" thing is....yet...just wait)

Let's return to the limit definition of a derivative for $f(x) = 2^x$:

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{2^{x+h}-2^x}{h}$$

$$= \lim_{h\to 0}\frac{2^x(2^h-1)}{h}$$ ← remove common factor of 2^x

$$= 2^x\left(\lim_{h\to 0}\frac{(2^h-1)}{h}\right)$$

...we also know that $f'(x) = 2^x \ln 2$....therefore

$$\lim_{h\to 0}\frac{(2^h-1)}{h} = \ln 2$$

similarly:

$$\lim_{h\to 0}\frac{(3^h-1)}{h}$$
$=$

$$\lim_{h\to 0}\frac{(7^h-1)}{h}$$
$=$

$$\lim_{h\to 0}\frac{\left(\left(\frac{1}{2}\right)^h-1\right)}{h}$$
$=$

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General Results: Where $f(x) = a^x$

1. When $a = e$, $f'(x) = f(x)$
2. $f'(x) = a^x \ln a$
3. $\ln a = \lim_{h \to 0} \frac{(a^h - 1)}{h} = \frac{f'(x)}{f(x)} = k$

Ex. 1 Show that the following are equivalent expressions.

$a)$ $\lim_{h \to 0} \frac{25^h - 1}{h}$ $b)$ $\lim_{h \to 0} \frac{5^{2h} - 1}{h}$ $c)$ $\lim_{h \to 0} \frac{(5^h - 1)(5^h + 1)}{h}$

Ex. 2 Determine the derivative of each function by first principles.

(...try to predict the answer based on your ever expanding knowledge of derivative functions)

a) $f(x) = 2^{x+3}$

b) $f(x) = e^{x}$

c) $f(x) = 5^{2x}$

p. 233 #14, 15

p. 240 #1acf, 4,5 **all from first principles